SPECTRUM SHARING IN COGNITIVE RADIO SYSTEMS UNDER OUTAGE PROBABILITY CONSTRAINT

A Thesis

by

PEI LI CAI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2009

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Shuguang Cui
Committee Members,	Henry D. Pfister
	Peng Li
	J. Maurice Rojas
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ABSTRACT

Spectrum Sharing in Cognitive Radio Systems under Outage Probability Constraint. (December 2009)

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Chair of Advisory Committee: Shuguang Cui

For traditional wireless communication systems, static spectrum allocation is the major spectrum allocation methodology. However, according to the recent investigations by the FCC, this has led to more than 70% of the allocated spectrum in the United States being under-utilized. Cognitive radio (CR) technology, which supports opportunistic spectrum sharing, is one idea that is proposed to improve the overall utilization efficiency of the radio spectrum.

In this thesis we consider a CR communication system based on spectrum sharing schemes, where we have a secondary user (SU) link with multiple transmitting antennas and a single receiving antenna, coexisting with a primary user (PU) link with a single receiving antenna. At the SU transmitter (SU-Tx), the channel state information (CSI) of the SU link is assumed to be perfectly known; while the interference channel from the SU-Tx to the PU receiver (PU-Rx) is not perfectly known due to less cooperation between the SU and the PU. As such, the SU-Tx is only assumed to know that the interference channel gain can take values from a finite set with certain probabilities. Considering a SU transmit power constraint, our design objective is to determine the transmit covariance matrix that maximizes the SU rate, while we protect the PU by enforcing both a PU average interference constraint and a PU outage probability constraint. This problem is first formulated as a non-convex optimization problem with a non-explicit probabilistic constraint, which is then approximated as a mixed binary integer programming (MBIP) problem and solved with the Branch and Bound (BB) algorithm. The complexity of the BB algorithm is analyzed and numerical results are presented to validate the effectiveness of the proposed algorithm. A key result proved in this thesis is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under PU outage constraints. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

To My Family

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My graduate studies at Texas A&M University have been some of the most memorable times of my life. During this period I have matured both intellectually and spiritually. First and foremost, I would like to express my deepest gratitude toward my parents. They have dedicated their entire lives for my growth. All I have achieved today is a result of their boundless support and encouragement.

I'm also extremely grateful to my advisor, Prof. Shuguang Cui, who gave me a chance to study electrical engineering when I didn't have much engineering background. His dedication and guidance have truly been an inspiration. Working with Prof. Cui has helped me gain valuable insights in both my research and life.

I would also like to thank all of my committee members, Dr. Peng Li, Dr. Henry Pfister, and Dr. J. Maurice Rojas, for their advice and support in my studies and during my thesis preparation.

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Lastly, I thank my family and girlfriend, Ruby, who have always supported me during my studies.

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CHAPTER I

INTRODUCTION

Wireless technology has experienced tremendous growth in the communication industry over the past 10 years. For example, cellular systems have grown and become such an integrated part of our daily lives, and cell phones have completely replaced their land-line counterparts in many modern households. In addition, wireless local area networks (WLAN) have also become ubiquitous in many homes, college campuses, and work places, where they play a vital role in increasing simplicity, flexibility, and the overall productivity. Due to the rapid pace of development in the field of wireless communications, many new applications and services are expected to accommodate the ever-increasing wireless needs, which includes mobile tv, real-time target tracking, and telecommerce.

With the emergence of a vast number of wireless services and applications as described above, the electromagnetic radio spectrum is becoming more and more crowded in order to support legacy systems as well as newly developed technologies [1]. In many countries, the radio spectrum is typically controlled and regulated by government agencies, where frequency bands are sold or licensed, often at exorbitant prices to private operators for exclusive commercial uses. However, recent studies have found that a large portions of the radio spectrum is significantly under-utilized. As a result, the cognitive radio technology [2], [3] has been proposed to improve the efficiency in spectrum utilization by exploiting frequencies that are not being used by licensed users at a given time and location.

In this chapter, we first give an introduction to the cognitive radio technology in

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The journal model is *IEEE Transactions on Automatic Control*.

Section A, which includes a brief history of cognitive radio, definitions of key terms, and a short summary of research results that have appeared so far. Then in Section B, we present an overview of the contributions of this thesis.

A. Overview of Cognitive Radio Technology

1. Motivation

The idea of cognitive radio (CR) was first proposed in 1999 by Joseph Mitola et al. [2], as an advanced version of software radio. Ideally, cognitive radios are able to communicate with each other over any available frequency spectrum where the frequency band is not being used by other users at a given time and location. The question then is where and when do we look for unused spectrum and how do we utilize it.

Currently radio spectrum is regulated by government agencies with essentially two models of allocation and usage. Either the spectrum is sold or licensed to private operators such as Verizon, or AT&T, etc.; or the spectrum is open to public for anyone to use upon an agreement of certain access rules, e.g., in WLAN and bluetooth systems. In most developed countries, the first model is the predominant choice despite the fact that it is highly inefficient in terms of frequency utilization. In fact, by scanning the spectrum across rural and urban geographical locations, we would find that though some frequency bands are heavily used, many bands are only partially used or mostly unused at a given time. According to a recent study done by New American Foundation and Shared Spectrum Company [4], in the 30-300MHz range, the utilization of some of the radio channels is less than one percent; and in the continuous range of frequencies between 30MHz and 3GHz, the estimated average usage during peak hours in an urban setting is only 38%, which suggests that even during peak hours free spaces in the spectrum, or spectrum holes, can be found. We therefore conclude that a rigid and static allocation of spectrum in blocks could be one of the main reasons for the current frequency scarcity issue.

Clearly, spectrum utilization can be significantly improved by having a cognitive secondary user accessing “spectrum holes” that are not being used by primary users at a specific time and location. This is one of the main motivations behind the current research efforts in the design and implementation of cognitive radios. With the above discussion in mind, we present the following definition for cognitive radio provided by the author of [5]:

Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind: 1. highly reliable communications whenever and wherever needed; 2. efficient utilization of the radio spectrum.

In the following section, we present an overview of the existing results related to our efforts on the CR design problems.

2. Challenges and Prior Work

Thanks to the significant advances over the past decade in software, hardware, digital signal processing, and computer networking, the realization of cognitive radio with the combination of aforementioned elements is indeed feasible today or in the near future. Nevertheless, there are many fundamental challenges ahead in the design and implementation of CR systems. In general, a CR sysstem consists of primary users (PUs) who have priority in utilizing the spectrum resource, and secondary users (SUs) who only access or share the spectrum opportunistically. That means the SUs can only share the frequency spectrum with the PUs when certain quality-of-service (QoS) of the PUs is guaranteed.

One natural challenge here is how to design a secondary system to simultaneously maintain the QoS of the primary users while maximizing its own achievable throughput. In particular, to safe-guard the QoS of the primary users in a cognitive radio setting, the key is to control the real-time interaction between the SU transmitters and the PU receivers. This leads to the definition of a new metric called *interference-temperature* [5], which is used to quantify the amount of SU interference allowed at a particular PU receiver (PU-Rx). Mathematically, this leads to certain interference power constraints at the SU transmitters (SU-Tx).

Under such a setup, much work has been devoted to analyzing the performance of cognitive radio systems. The author in [6] studied the channel capacity of a single secondary transmission with interference temperature constraints at the PU-Rx as opposed to the conventional transmit power constraint. The capacities of different additive white Gaussian noise (AWGN) channels were derived. The authors in [7] studied the ergodic capacity of the secondary link with different fading scenarios under average or the instantaneous interference-power constraint. The conclusion is that significant capacity gains may be achieved when the channels experience fading. Along a similar line, the authors in [8] studied the problem of multiple SUs and maximizing their sum utilities using game theory under the interference temperature constraints, where a secondary spectrum sharing potential game was defined, and a sequential play solution and a stochastic learning algorithm were proposed, which converges to the Nash equalibrium. A more information-theoretic approach was taken in [9], where the authors studied the achievable rate region of a genie-aided CR channel, in which the sender is non-causally given the messages of others. More studies on the performance of cognitive radio systems can also be found in [10] - [12].

Wireless communication via multiple transmit and receive antennas has drawn much attention in the past decade. In general, multi-antennas can be utilized to improve performance through diversity, or increase data rate by multiplexing. Basically, multi-antennas can provide more degrees of freedom in space in addition to those in time and frequency, and thus provide an increase in spectral efficiency. Therefore, it is logical to explore the role of multi-antennas in a cognitive radio setting. The authors in [13] studied the channel capacity of secondary multiple-input multiple-output (MIMO) and multiple-input single-output (MISO) channels when the CSI between the SU-Tx and the PU-Rx is perfectly known at the SU-Tx. In the MISO case, under both an average secondary transmit power constraint and an interference temperature constraint at each PU-Rx, beamforming was proved to be the optimal. In the MISO case where only one PU is present with one receiving antenna, a closed-form solution was derived. In [14] and [15], the authors considered a similar MISO scenario where, instead of the complete CSI between the SU-Tx and the PU-Rx, only partial CSI is known. In [14], channel capacity was studied with only the mean of the channel between the SU-Tx and the PU-Rx is known at the SU-Tx, where beamforming was proven optimal. Such work was extended in [15] to consider both the mean and covariance feedbacks at the SU-Tx, where two algorithms are presented to solve for the optimal solution: one based on a second-order cone programming approach; and the other based on a geometric interpretation.

In our work, we only assume that the SU-Tx knows the distribution of the channel to the PU-Rx, and solve for the optimal transmission strategy under the MISO setup. We next summarize our main contributions.

B. Overview of Contributions

In this thesis, we model a scenario where we know the distribution of the SU-Tx to PU-Rx channel and formulate the problem under a PU outage probability constraint in addition to the transmit power constraint and the average interference power constraint. In our work, we define the outage probability to be the probability of interference power at the PU-Rx exceeding a given threshold. The main motivation for this formulation is to allow some interference from the SU-Tx to the PU-Rx as long as the resulting outage probability is kept small. Our aim in this thesis is to investigate the SU system performance with the above practical regulation over the SU interference to the PU-Rx.

The main contribution of this thesis is summarized as follows. We formulate the transmit covariance matrix design problem for a single secondary link under both an average interference constraint and an outage probability constraint to protect a given PU-Rx. Due to the introduction of the outage probability constraint, this resulting design problem is non-convex with non-explicit constraints. To solve this problem, we reformulate it into a MBIP problem with a deterministic constraint on the outage upper bound. Then we adopt two different approaches: a branch and bound algorithm to compute the globally optimal solution, and a heuristic method to find an efficient but suboptimal solution. A key result proved in this thesis is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under PU outage constraints. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

This thesis is structured as follows. In Chapter II, we begin in Section II with a background introduction for convex optimization, which is widely used in our work.

In Chapter III, we fist give a detailed description of the cognitive radio problem under consideration with a probabilistic constraint. Then we present the signal and system model, and discuss the initial formulation of the non-convex problem. In the following sections, we discuss the details of an approximation transformation of the non-convex problem into a mixed binary integer program. Since the complexity of solving the MBIP through exhaustive search is high in general, we propose a branch and bound algorithm as well as a heuristic algorithm to solve this problem in an efficient manner. At last, we present the complexity analysis and numerical results. In Chapter IV, we conclude with a summary of contributions and possible future works.

CHAPTER II

CONVEX OPTIMIZATION AND ALGORITHMS

Since convex optimization techniques are widely used in our work, in this section we give a short background overview for the self-completeness of this thesis. The mathematics of convex optimization has been studied for almost a century. Some recently developments have sparked renewed interest in this area, namely the generalization of interior-point methods and the expanded applications of convex optimization.

Although interior-point methods were first developed in the 1980s [16] to solve linear programming problems, in recent years, they have been generalized to solve other convex optimization problems as well. This means that we now have the tools to solve semi-definite programming problems, second-order cone programming problems, and other classes of convex optimization problems, almost as easily as solving linear programs. Furthermore, researchers discovered that the application of convex optimization extends far beyond what people initially thought. For example, in the last two decades, convex optimization have found extensive applications in control theory [17], signal processing [18], circuit design [19], statistics [20], and even finance [21] and so on. We believe that there will be much more applications of convex optimization to be explored as research in this area continues to expand.

The reason behind its extensive applications is obvious: there are many benefits of recognizing and formulating a problem as a convex optimization problem. One of the major advantages is that convex optimization problems can be solved numerically in an efficient and reliable manner through the use of interior-point methods, although sometimes other special methods can also be applied. Also, in some special cases, an analytical solution may be obtained in the closed-form by applying the the Karush-Kuhn-Tucker (KKT) optimality conditions [29]. Moreover, from either a theoretical or conceptual point of view, interesting interpretations or insights can often be observed through the use of duality theories [29].

The remainder of this chapter is structured as follows. A general overview of convex analysis will first be given, where we discuss some basic mathematical concepts behind the theories of convex sets and functions. Then we introduce basic optimization terminologies, followed by a few common convex optimization problems and their properties. Lastly, we present the duality theory and KKT conditions, which are necessary and sufficient for the solution optimality in convex optimization problems.

A. Convex Analysis

Convex optimization problems is a class of optimization problems which has found tremendous number of applications mainly in engineering, finance, and statistics. In the following sections we will first introduce some basic definitions in the theory of convex analysis, followed by optimization problems and examples. Since convex optimization techniques are widely used in our work, in this section we give a short background overview for the self-completeness of this thesis[1].

1. Convex Sets and Functions

a. Convex Set

A set D is defined to be *convex* if and only if between any two points in D, the line segment connecting the two points also lies in D, *i.e.,* if and only if for any x_1, $x_2 \in D$

[1] Note that most of the material in this chapter are based on the textbook of Convex Optimization [29]. We thank the authors for their tremendous contributions in popularizing the concepts of convex optimization among engineering students.

and any θ within $[0, 1]$, we have

$$\theta x_1 + (1 - \theta)x_2 \in D. \tag{2.1}$$

Informally speaking, a set is convex if every point in the set can be connected to every other point in the set by a line segment that is contained entirely in the set itself, where Fig. 1 illustrates two simple examples of this idea in $\mathbf{R}^2$.

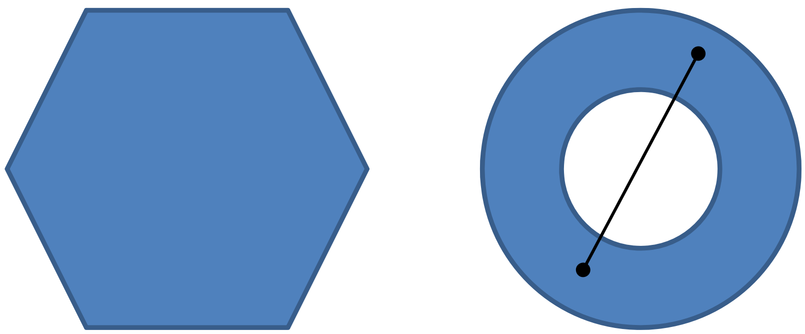

Fig. 1. Two simple sets, one convex and one non-convex. *Left.* The hexagon, including its boundaries, is convex. *Right.* The donut-shaped set is not convex since the line segment between the chosen two points is not contained entirely in the set.

In our work, we will frequently use a particular convex set: the set of positive semidefinite matrices. In recent studies, it has been discovered that such a set plays an important role in many applications of convex optimization. In the rest of this thesis, we use the notation $\mathbf{S}^n$ to denote the set of symmetric $n \times n$ matrices,

$$\mathbf{S}^n = \{\mathbf{X} \in \mathbf{R}^{n \times n} | \mathbf{X} = \mathbf{X}^T\}, \tag{2.2}$$

which is an $n(n+1)/2$ dimensional vector space. Similarly, the notation $\mathbf{S}^n_+$ is used to denote the set of symmetric positive semi-definite matrices,

$$\mathbf{S}_{+}^{n} = \{\mathbf{X} \in \mathbf{S}^{n} | \mathbf{X} \succeq 0\}, \tag{2.3}$$

and $\mathbf{S}_{++}^{n}$ denotes the set of symmetric positive definite matrices,

$$\mathbf{S}_{++}^{n} = \{\mathbf{X} \in \mathbf{S}^{n} | \mathbf{X} \succ 0\}. \tag{2.4}$$

The above sets: $\mathbf{S}^{n}$, $\mathbf{S}_{+}^{n}$, and $\mathbf{S}_{++}^{n}$ are all convex, which can be proved directly from their respective definitions. Especially, $\mathbf{S}_{+}^{n}$ is also called the positive semidefinite (SDP) cone.

b. Convex Functions

A function $f : \mathbf{R}^{n} \to \mathbf{R}$ is *convex* if and only if its domain, $\mathbf{dom}\ f$, is a convex set and for all $\mathbf{x}, \mathbf{y} \in \mathbf{dom}\ f$, and θ within $[0, 1]$, we have the following inequality:

$$f(\theta\mathbf{x} + (1-\theta)\mathbf{y}) \leq \theta f(\mathbf{x}) + (1-\theta) f(\mathbf{y}). \tag{2.5}$$

Geometrically, the inequality can be interpreted as that the line segment, or the *chord* between any two points of the graph of f, lies above the graph of f. This is shown in Fig. 2. Similarly, a function is *concave* if $-f$ is convex.

2. Convex Optimization Problems

In this section, we will first present the basics of optimization problems including standard terminology, and some useful properties. We then focus our attention on convex optimization problems and some of its subclasses such as linear programming(LP) problems, and semidefinite programming (SDP) problems. Lastly, we will introduce the Lagrangian dual function and its importance to deriving the Karush-Kuhn-Tucker (KKT) conditions for optimality.

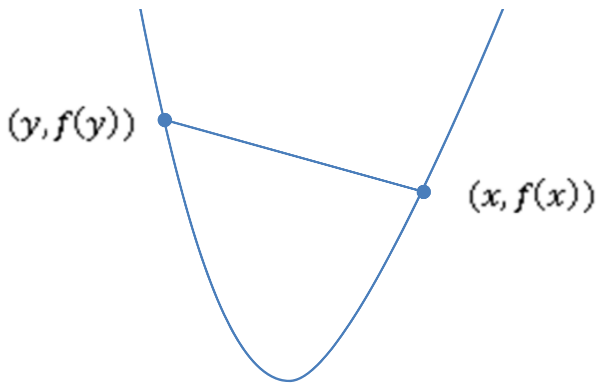

Fig. 2. The graph of a convex function where the chord, between two arbitrary points, lies above the function.

a. Optimization Terminology

We first present the *standard form* of an optimization problem as follows:

$$\text{minimize:} \quad f_0(\mathbf{x}) \tag{2.6}$$

$$\text{subject to:} \quad f_i(\mathbf{x}) \leq 0, \quad i = 1, ..., m \tag{2.7}$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, ..., p. \tag{2.8}$$

This notation can be interpreted as finding an $\mathbf{x}$ among all $\mathbf{x}$'s such that $f_0(\mathbf{x})$ is maximized, while the conditions $f_i(\mathbf{x}) \leq 0,\ i = 1, ..., m$ and $h_i(\mathbf{x}) = 0,\ i = 1, ..., p$ are satisfied. Here, we call $\mathbf{x} \in \mathbf{R}^n$ the *optimization variable*, or *design variable*, and the function $f_0 : \mathbf{R}^n \rightarrow \mathbf{R}$ the *objective function*. The inequalities in (2.7) are called *inequality constraints* with corresponding *inequality constraint functions* $f_i : \mathbf{R}^n \rightarrow \mathbf{R}$; and the equations in (2.8) are called *equality constraints* with corresponding *equality constraint functions* $h_i : \mathbf{R}^n \rightarrow \mathbf{R}$. If there is no inequality (2.7) and equality (2.8) constraints in the optimization problem, the problem is called *unconstrained*.

The *domain* of the optimization problem is the set of points over which the objective function and the constraint functions are defined, denoted by

$$\mathcal{D} = \bigcap_{i=0}^{m} \textbf{dom } f_i \cap \bigcap_{i=1}^{p} \textbf{dom } h_i. \tag{2.9}$$

A point $\mathbf{x} \in \mathcal{D}$ is called *feasible* if it satisfies both the inequality constraints in (2.7) and the equality constraints in (2.8). The set of all feasible points together is called the *feasible set*. If the feasible set contains at least one point, the optimization problem is said to be *feasible*; if the feasible set is the empty set, the optimization problem is called *infeasible*. The *optimal value* of the above optimization problem is defined as

$$p^* = \inf \{f_0(\mathbf{x}) \mid f_i(\mathbf{x}) \leq 0,\ i = 1, ..., m,\ h_i(\mathbf{x}) = 0, i = 1, ..., p\}, \tag{2.10}$$

from which we define $\mathbf{x}^*$ as an *optimal point* if $\mathbf{x}^*$ is feasible and $f_0(\mathbf{x}^*) = p^*$.

b. Equivalent Problems

Two problems are called *equivalent* if from one problem the solution of the other can be easily obtained, and vice versa. This definition is an informal one, but it provides a good intuition of equivalence in optimization problems, which is important since it gives us the flexibility of solving an equivalent but simpler problem if the original problem is not nicely structured. For more details, please refer to Chapter 4 in [29].

c. Convex Optimization Problems in Standard Form

Having defined the standard form of a general optimization problem and its terminologies, we now present the standard *convex optimization problem*, which has the following form

$$\begin{aligned} \text{minimize:} \quad & f_0(\mathbf{x}) \\ \text{subject to:} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, ..., m_1 \\ & \mathbf{a}_i^T \mathbf{x} = b_i \quad i = 1, ..., p, \end{aligned} \tag{2.11}$$

where $f_0, ..., f_m$ are convex functions. Compared to the general optimization problem, the convex problem has three additional requirements: the objective function must be convex, the inequality constraint functions must be convex, and the equality constraint functions must be affine. One immediate observation is that the feasible set of a convex optimization problem is convex, since it is the intersection of m convex sublevel sets $\{\mathbf{x} \mid f_i(\mathbf{x}) \leq 0\}$, and p hyperplanes $\{\mathbf{x} \mid \mathbf{a}_i^T \mathbf{x} = b_i\}$. Thus, in a convex optimization problem, the objective is to minimize a convex function over a convex set.

One of the fundamental properties of convex optimization problems is that there is no distinction between locally and globally optimal points. Any locally optimal point is also globally optimal. This is one of the key reasons that it is advantageous to formulate problems as convex optimization problems. In the next two subsections, we introduce two subclasses of convex optimization problems, which will appear in our work.

d. LP and SDP Problems

LP Problems: A convex optimization problem is called a *linear programming problem* (LP) if the objective and constraint functions are all affine, which has the general form of

$$\begin{aligned} \text{minimize:} \quad & \mathbf{c}^T\mathbf{x} + d \\ \text{subject to:} \quad & \mathbf{Gx} \preceq \mathbf{h} \\ & \mathbf{Ax} = \mathbf{b}, \end{aligned} \tag{2.12}$$

where $\mathbf{G} \in \mathbf{R}^{m\times n}$ and $\mathbf{A} \in \mathbf{R}^{p\times n}$. There are two forms of LP that are frequently encountered. In the *standard form* LP, the component-wise nonnegative constraints are its only inequalities, that is

$$\begin{aligned} \text{minimize:} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to:} \quad & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \succeq 0. \end{aligned} \tag{2.13}$$

If the LP has no equality constraints, then it is called an *inequality form* LP, usually formulated as

$$\begin{aligned} \text{minimize:} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to:} \quad & \mathbf{Ax} \preceq \mathbf{b}. \end{aligned} \tag{2.14}$$

Note that the constant d in the objective function is often omitted, since it has no affect on the optimal set of the problem.

SDP Problems: A *semidefinite programming* (SDP) problem has the form

$$\begin{aligned} \text{minimize:} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to:} \quad & x_1\mathbf{F_1} + ... + x_n\mathbf{F_n} + \mathbf{G} \preceq \mathbf{0} \\ & \mathbf{Ax} = \mathbf{b}, \end{aligned} \tag{2.15}$$

where $\mathbf{G}, \mathbf{F_1}, ..., \mathbf{F_n} \in \mathbf{S}^k$, and $\mathbf{A} \in \mathbf{R}^{p\times n}$, and the first inequality constraint is called a *linear matrix inequality* (LMI). Notice, if $\mathbf{G}, \mathbf{F_1}, ..., \mathbf{F_n}$ are all diagonal matrices, then the LMI in (2.15) reduces to a set of n linear inequalities, and the problem reduces to a linear program. It has been found that a large number of practical problems in circuit design, operations research, and control theory can be formulated or approximated as SDP problems, which can then be efficiently solved by the interior point methods.

Similar to LP, there are usually two forms of representing a SDP problem. The first is the *standard form* SDP with a linear equality constraint and a positive semidefinite constraint on the design variable $\mathbf{X} \in \mathbf{S}^n$:

$$\begin{aligned} \text{minimize:} \quad & tr(\mathbf{CX}) \\ \text{subject to:} \quad & tr(\mathbf{A_i X}) = b_i, \quad i = 1, ..., p \\ & \mathbf{X} \succeq 0, \end{aligned} \tag{2.16}$$

where $\mathbf{C}, \mathbf{A_i}, ..., \mathbf{A_p} \in \mathbf{S}^n$. Both LP and SDP in standard forms, we minimize a linear objective function, subject to p linear equality constraints, and a nonnegativity constraint on the variable. An *inequality form* SDP is given as:

$$\begin{aligned} \text{minimize:} \quad & \mathbf{c}^T \mathbf{x} \\ \text{subject to:} \quad & x_1 \mathbf{A_1} + ... + x_n \mathbf{A_n} \preceq \mathbf{B} \end{aligned} \tag{2.17}$$

with variable $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{B}, \mathbf{A_1}, ..., \mathbf{A_n} \in \mathbf{S}^k$, and $\mathbf{c} \in \mathbf{R}^n$. The above two forms could be equivalently transformed from one to the other.

e. Duality

The concept of duality [29] in convex optimization plays a key role in analyzing the optimal solutions structures. We begin by first considering a standard optimization problem

$$\begin{aligned} \text{minimize:} \quad & f_0(\mathbf{x}) \\ \text{subject to:} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, ..., m \\ & h_i(\mathbf{x}) = 0, \quad i = 1, ..., p, \end{aligned} \tag{2.18}$$

with design variable $\mathbf{x} \in \mathbf{R}^n$ and its domain $\mathcal{D} = \bigcap_{i=0}^{M} \textbf{dom } f_i \ \cap \ \bigcap_{i=1}^{p} \textbf{dom } h_i$. Without assuming convexity of the problem, we denote the optimal value of (2.18) by p^*. We define the *Lagrangian* $L : \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^p \to \mathbf{R}$ associated with (2.18) as

$$L(\mathbf{x}, \lambda, \nu) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}). \tag{2.19}$$

with $\textbf{dom } L = \mathcal{D} \times \mathbf{R}^m \times \mathbf{R}^p$, where $\lambda_i \geq 0$ and ν_i are referred to as the *Lagrange multipliers* associated with the ith inequality constraint $f_i(\mathbf{x}) \leq 0$, and the ith equality constraint $h_i(\mathbf{x}) = 0$, respectively. Together, we refer to vectors λ and ν as *dual variables* or *Lagrange multiplier vectors* associated with the problem (2.18).

Using the Lagrangian, we can define the *Lagrange dual function* $g : \mathbf{R}^m \times \mathbf{R}^p \to \mathbf{R}$ as the the minimum of the Lagrangian over the variable $\mathbf{x}$

$$g(\lambda, \nu) = \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \lambda, \nu) = \inf_{\mathbf{x} \in \mathcal{D}} \left(f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}) \right) \tag{2.20}$$

with $\lambda \succeq 0$, $\lambda \in \mathbf{R}^m$, and $\nu \in \mathbf{R}^p$. It is worth noting that the dual function is always concave even when (2.18) is not convex, since the dual function is the point-wise infimum of a family of affine functions in (λ, ν). An important property of the dual function is that it always yields a lower bound on the optimal value p^* of (2.18), i.e., the weak duality [29]:

$$g(\lambda, \nu) \leq p^*, \tag{2.21}$$

for any $\lambda \succeq 0$ and ν. This property naturally leads to the question of what is the best lower bound that can be obtained from the Lagrange dual function? This question is answered by the *Lagrange dual problem*:

$$\begin{aligned} \underset{\lambda,\nu}{\text{maximize:}} \quad & g(\lambda, \nu) \\ \text{subject to:} \quad & \lambda \succeq \mathbf{0}. \end{aligned} \tag{2.22}$$

Since this is the dual problem of (2.18), the original problem is often called the *primal problem*. Also, we refer to (λ^*, ν^*) as *dual optimal* or *optimal Lagrange multipliers* if they are optimal for the dual problem (2.22). As previously mentioned, problem (2.18) is not necessarily convex. However, the dual problem (2.22) must be a convex optimization problem since we are maximizing a concave objective function over convex constraints.

f. Karush-Kuhn-Tucker Optimality Conditions

When the primal problem (2.18) is convex and strictly feasible, we have the strong duality [29]:

$$g(\lambda^*, \nu^*) = p^*, \tag{2.23}$$

which leads to the following necessary and sufficient optimality conditions for λ^*, ν^*, and $\mathbf{x}^*$, i.e., the Karush-Kuhn-Tucker (KKT) conditions [29]:

$$\begin{aligned} f_i(\mathbf{x}^*) \leq 0, \quad & i = 1, ..., m \\ h_i(\mathbf{x}^*) = 0, \quad & i = 1, ..., p \\ \lambda_i^* \geq 0, \quad & i = 1, ..., m \\ \lambda_i^* f_i(\mathbf{x}^*) = 0, \quad & i = 1, ..., m \\ \nabla f_0(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* \nabla f_i(\mathbf{x}^*) + \sum_{i=1}^{p} \nu_i^* \nabla h_i(\mathbf{x}^*) = 0. \quad & \end{aligned} \tag{2.24}$$

The KKT conditions are an important in convex optimization for two main reasons: In some special cases when it is possible to directly solve for the KKT conditions, it offers the optimal solution in a closed form; even in general cases, the KKT conditions serve as the principle components of many numerical algorithms designed to solve the convex optimization problem.

In the first part of this chapter, we presented some basic definitions on convex sets and functions. In the second part, we formally introduced convex optimization problems, relevant terminologies, and widely encountered problems including linear programs, and semidefinite programs. Also, we introduced the important concept of duality and the role that it plays in solving convex optimization problems, which leads to the (KKT) conditions for optimality.

CHAPTER III

COGNITIVE SPECTRUM SHARING WITH OUTAGE PROBABILITY CONSTRAINT

As we discussed previously, the evolution from static spectrum allocation policies to dynamic ones can significantly increase the utilization efficiency of the radio spectrum. One promising platform to support such transitions is the CR system that was invented for opportunistic spectrum sharing with existing primary links, where CRs dynamically adapt their transmission patterns to access under-utilized frequency segments while regulating the interference to PUs [2], [5]. As such, the key design challenge is how to maximize the SU rate while maintaining an acceptable level of interference to PUs.

In this thesis, we model a practical scenario where we know the distribution of the SU-Tx to PU-Rx channel and formulate the problem under an PU outage probability constraint in addition to the transmit power constraint and the average interference power constraint. In our work, we define the outage probability to be the probability of interference power at the PU-Rx exceeding a given threshold. The main motivation for this formulation is to allow some interference from the SU-Tx to the PU-Rx as long as the resulting outage probability is kept small. Our aim in this thesis is to investigate the SU system performance with this more practical regulation over the SU interference to the PU-Rx.

The main contribution of our work is summarized as follows. We formulate the transmit covariance matrix design problem for a single secondary link under both an average interference constraint and an outage probability constraint to protect a given PU-Rx. Due to the introduction of the outage probability constraint, this resulting design problem is non-convex with non-explicit constraints. To solve this problem, we reformulate it into an MBIP problem with a deterministic constraint on the outage upper bound. Then we use a BB algorithm to compute the numerical results, which is highly efficient in solving the MBIP problem compared with exhaustive searching for the original non-convex problem. A key result proved in this paper is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under PU outage constraints. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

The rest of the chapter is organized as follows. In Section A, we discuss the system and signal models. In Section B, the MBIP transformation is discussed along with the BB algorithm and the complexity analysis, and we show that the rank of the optimal transmit covariance matrix is always one. In addition, we also propose a simple algorithm as an alternative to the BB algorithm for finding a good suboptimal solution to the MBIP problem. In Section C, the numerical results are presented.

Notations: $\mathbf{x}^{\dagger}$ denotes the conjugate transpose, $tr(\cdot)$ denotes the trace operator, $rank(\cdot)$ denotes the rank of a matrix, $E[\cdot]$ denotes the statistical expectation, and $C^{M\times N}$ denotes the space of $M \times N$ matrices with complex entries. Boldface upper and lower case letters are used to denote matrices and vectors, respectively, with "$\sim$" standing for "distributed as". $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real and imaginary parts of the operand, respectively. The $\log(\cdot)$ functions are over base 2.

A. System and Signal Model

We consider a simple CR system, where one SU link and one PU link share the same spectrum simultaneously. Here the SU-Tx is equipped with M_t transmitting antennas, and both the secondary and primary receivers are each equipped with a single antenna, as illustrated in Fig. 3. We assume that the SU-Tx knows the MISO channel $\mathbf{h} \in C^{M_t \times 1}$ from the SU-Tx to the SU-Rx, which is randomly distributed according to $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{I})$. The MISO interference channel from the SU-Tx to the PU-Tx, denoted as $\mathbf{g} \in C^{M_t \times 1}$, is not perfectly known to the SU-Tx due to less cooperation between the SU and the PU. Specifically, we assume that the SU-Tx knows that the interference channel $\mathbf{g}$ can take values from a finite set $\mathbf{G} = \{\mathbf{g_1}, \mathbf{g_2}, \cdots, \mathbf{g_N}\}$ with a corresponding probability set $\{p_1, p_2, \cdots, p_N\}$. Under these assumptions, the SU-Tx adapts the transmission rate, power, and spatial spectrum to maximize its own transmission rate, while maintaining the interference to the PU-Rx below a certain level. Such an interference regulation is achieved by enforcing a set of constraints over the SU transmit covariance matrix, which will be discussed later in details. The signal model for the system under consideration is given as

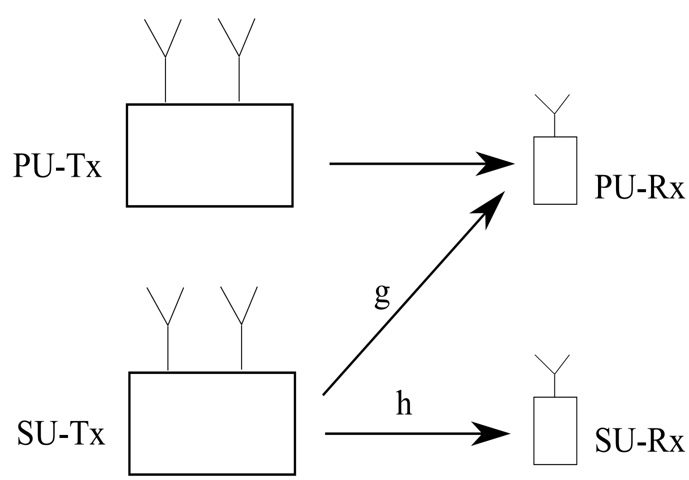

Fig. 3. System model of the SU coexisting with the PU.

$$y = \mathbf{h}^{\dagger}\mathbf{x} + w, \tag{3.1}$$

where y and $\mathbf{x} \in C^{M_t \times 1}$ are the received and transmitted signals at the SU-Rx and SU-Tx, respectively, and w is the additive Gaussian noise with $w \sim \mathcal{CN}(0,1)$. The transmit covariance matrix is denoted by $\mathbf{K_x} = E[\mathbf{x}\mathbf{x}^\dagger] \succeq 0$.

Our goal in this thesis is to balance the maximum transmit rate of the SU and the interference from the SU-Tx to the PU-Rx by adjusting the spatial spectrum of the SU signal. As such, we need to design the optimal transmit covariance matrix, $\mathbf{K_x}$, to maximize the SU rate with some tolerable interference to the PU-Rx. In particular, we cast this problem as the following (P1):

$$\underset{\mathbf{K_x}}{\text{maximize:}} \quad \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h} \tag{3.2}$$

$$\text{subject to:} \quad tr(\mathbf{K_x}) \leqslant P_{tr1} \tag{3.3}$$

$$E[\mathbf{g}^\dagger \mathbf{K_x} \mathbf{g}] \leqslant P_{tr2} \tag{3.4}$$

$$Pr\{\mathbf{g}^\dagger \mathbf{K_x} \mathbf{g} \geqslant r\} \leqslant p_{th} \tag{3.5}$$

$$\mathbf{K_x} \succeq 0. \tag{3.6}$$

where the objective is equivalent to maximizing the achievable rate $\log(1 + \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h})$, P_{tr1} is the SU transmit power limit, P_{tr2} is the average interference power limit, r is the instantaneous interference power tolerance at the PU-Rx, and p_{th} is the PU outage probability limit. The objective function is the SU transmission rate, and the four constraints are the average transmit power, the average interference power, the PU outage probability constraints, and the positive semi-definite constraint, respectively.

Due to the probabilistic constraint in (3.5), problem (P1) is generally hard to solve directly. For a probabilistic constraint where the random vector has a continuous distribution, checking the feasibility of each feasible point requires a complex multi-dimensional integration. Even when the random vector has a discrete distribution, the feasible set defined by the probabilistic constraint is generally non-convex and it cannot be described by explicit functions [22]. Fortunately, as shown in [23], [24], the above probabilistically constrained problems can be solved as integer programming (IP) problems with deterministic constraints.

For our problem, under the assumption that the SU-Tx knows that the interference channel $\mathbf{g}$ is of a finite discrete distribution, we take the approach in [23], [24] to first approximate (P1) as an MBIP problem with deterministic constraints, and then deploy a BB algorithm [25], [26], [27] to seek the solution. The details will be discussed in the next sections, together with complexity analysis and simulation results.

B. Optimization Algorithm

1. MBIP Transformation

In this subsection, we first discuss a deterministic transformation of the probabilistic constraint in (P1). As assumed, the random variable $\mathbf{g}$ takes values from a finite set $\mathbf{G} = \{\mathbf{g_1}, \mathbf{g_2}, \cdots, \mathbf{g_N}\}$ with a corresponding probability set $\{p_1, p_2, \cdots, p_N\}$. We refer to each probable value $\mathbf{g_n}$ as one scenario. The probabilistic constraint can then be interpreted as that the sum probability over all possible interference-violating scenarios must be bounded by p_{th}. Therefore, we can reformulate the probabilistic constraint in (P1) as shown in the following problem (P2):

$$\underset{\mathbf{K_x},\ b_n}{\text{maximize:}} \quad \mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h} \tag{3.7}$$

$$\text{subject to:} \quad tr(\mathbf{K_x}) \leqslant P_{tr1} \tag{3.8}$$

$$E[\mathbf{g}^{\dagger}\mathbf{K_x}\mathbf{g}] \leqslant P_{tr2} \tag{3.9}$$

$$\mathbf{g_n}^{\dagger}\mathbf{K_x}\mathbf{g_n} - Mb_n \leqslant r, \quad n = 1, 2, \cdots, N. \tag{3.10}$$

$$\sum_{n=1}^{N} b_n p_n \leqslant p_{th}, \quad b_n \in \{0, 1\} \tag{3.11}$$

$$\mathbf{K_x} \succeq 0. \tag{3.12}$$

The two newly added constraints (3.10) and (3.11) are deterministic and only involving explicit functions, which can be easily handled by numerical algorithms. The design variables here are now both the matrix $\mathbf{K_x}$ and the binary variables $b_n, n = 1, 2, \cdots, N$, where the binary variables are used to indicate whether the interference outage check needs to be performed: if $b_n = 0$, it means no outage is possible under the scenario $\mathbf{g_n}$ given the constraint (3.10), such that p_n needs not to be included in the left-hand sum of (3.11); if $b_n = 1$, there may or may not be an outage if the slack constant M is chosen large enough, which leads to the fact that (3.11) is enforcing an outage probability upper-bound to be less than p_{th} since p_n is now always counted in the left-hand sum of (3.11). The positive slack constant, M, is chosen to be of a large value since it is used to deactivate the outage check in (3.10) when $b_n = 1$. Given the fact that $\sum_{n=1}^{N} b_n p_n$ incurs an outage probability upper-bound, (P2) is actually a stricter version of (P1) with tighter constraints. As a result, the optimal objective value of (P2) will be slightly less than that of (P1). However, as we show later that the resulting performance is still much better than reference approaches.

We now discuss how to determine the value for M, which needs to guarantee the satisfaction of the inequality (3.10) when $b_n = 1$. For sufficiency, we could find an M that is larger than the maximum value of $\mathbf{g_n^\dagger K_x g_n}$ over $n = 1, ..., N$. One way to achieve that is as follows. Given that

$$\mathbf{g_n^\dagger K_x g_n} = tr(\mathbf{g_n^\dagger K_x g_n}) = tr(\mathbf{K_x g_n g_n^\dagger}) \leqslant tr(\mathbf{K_x}) tr(\mathbf{g_n g_n^\dagger}) \leqslant P_{tr1} tr(\mathbf{g_n g_n^\dagger}), \quad (3.13)$$

we take $M = \max_n P_{tr1} tr(\mathbf{g_n g_n^\dagger})$.

With the value of M available, we next solve the MBIP problem (P2), for which a direct approach is through exhaustive search over the binary variables b_n's, where for each feasible choice of b_n's we solve the following convex semi-definite programming (SDP) problem

$$\begin{aligned}
\underset{\mathbf{K_x}}{\text{maximize:}} \quad & tr(\mathbf{K_x h h^\dagger}) && (3.14)\\
\text{subject to:} \quad & tr(\mathbf{K_x}) \leqslant P_{tr1} && (3.15)\\
& E[tr(\mathbf{K_x g g^\dagger})] \leqslant P_{tr2} && (3.16)\\
& \mathbf{tr}(\mathbf{K_x g_n g_n^\dagger}) \leqslant M b_n + r, \quad n = 1, 2, \cdots, N. && (3.17)\\
& \mathbf{K_x} \succeq 0. && (3.18)
\end{aligned}$$

Unfortunately, such an exhaustive search in general incurs exponential total complexity. So instead, we discuss a BB approach to search over the binary variables more efficiently in the average sense.

2. Branch and Bound Algorithm

As mentioned before, one way to solve an MBIP problem is through exhaustive search, where the feasible space grows exponentially with the number of binary variables, which leads to the NP-hardness of most binary optimization problems. Fortunately, BB algorithms [25], [26], [27] can often be used in solving discrete and combinatorial optimization problems to reduce the average complexity, when the problem has a finite but very large solution set with certain structures.

We first give a brief overview of the BB algorithm, followed by specific implementations for solving the MBIP problem (P2). Two components are usually required for an effective implementation of a BB algorithm. The first is a *branching procedure* and the second is a *bounding function*. Given a set S, the branching procedure returns non-overlapping subsets $S_1, S_2, ...$, whose union is the set S. The bounding function then computes the upper and/or lower bounds of the optimal value given each subset S_i. The upper and lower bounds are then used to determine one of the following two outcomes: split the subset S_i into more subsets for further bounding, or discard the subset S_i from the searching space, which is also referred to as pruning and is the reason why the BB algorithm is more efficient than exhaustive search.

It is clear that problem (P2) can be cast as a SDP problem over the design variable $\mathbf{K_x}$ when the binary variables are relaxed to be within $[0, 1]$. With this property, we next implement the BB algorithm to jointly search over $\mathbf{K_x}$ and the binary variable b_n. Due to the recursive nature of the BB algorithm, it traverses a binary search tree (BST), as shown in Fig. 2. Each *node* in the BST represents a particular case when the relaxed SDP problem from (P2) is solved at a partial or complete binary solution. In particular, the root node corresponds to the case where all b_n's are relaxed to be within $[0, 1]$; and a *leaf node* is a node at the bottom of the BST, which denotes the case with a complete binary solution, where the resulting objective value of (P2) is called an *incumbent* if it is the best objective value found so far across all the known leaf nodes. The *depth* of a node, D, is defined to be the number of determined binary variables in the partial binary solution at this node. As D increases from $D = j$ to $D = j + 1$, one additional binary variable b_n is being determined. Specifically, at one particular node let us assume that $b_1, b_2, ..., b_{n-1}$ have been determined. We then create two child nodes corresponding to two sub-problems in the relaxed SDP form of (P2) with $b_n = 0$ and $b_n = 1$, respectively, while keeping $b_1, b_2, ..., b_{n-1}$ unchanged and rounding all undetermined binary variables, $b_{n+1}, b_{n+2}, ..., b_N$, to be ones. For a given sub-problem, if the achieved optimal objective value is lower than the current incumbent, the corresponding child node (as well as all of its descendants) is discarded, i.e., pruned from the searching space. Otherwise, the corresponding child node is kept in the BST, and the searching continues to b_{n+1} until we reach the leaf node with a complete binary solution. Following the above procedure, the BB algorithm traverses

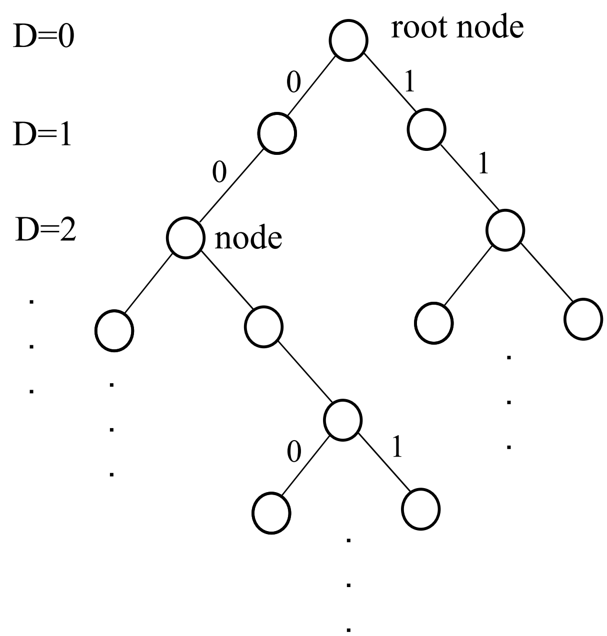

Fig. 4. Binary Search Tree (BST).

through the BST by solving one relaxed SDP for an optimal $\mathbf{K_x}$ at each node. The algorithm is terminated when the entire BST has been either pruned or processed. All computations in our algorithm are performed using the matlab-based software package CVX [28], [29] which deploys SeDuMi [30] as its back-end solver for SDP problems.

3. Complexity Analysis

In this subsection, we discuss the complexity of the proposed algorithm. The efficiency of the algorithm depends critically on the branching and bounding procedure, where the entire searching space is branched into non-overlapping subsets, and the bounding procedure then calculates bounds for each subset with decisions made on whether to continue branching or to discard the entire subset. The pruning process, which allows the algorithm to only traverse a fraction of the entire BST, is the key to decrease the overall searching complexity.

In our implementation, at the root node, there are no determined binary variables, i.e., all binary variables are relaxed. At each child node, one additional binary variable is determined. During each iteration, one node is chosen and the bound is calculated after solving the relaxed SDP. If the bound is lower than the incumbent, then it means that no child nodes branched from this node will yield a solution better than the incumbent; the node is therefore pruned. If the node at depth j is pruned, we can calculate how many potential child nodes of this branch are pruned, which indicates how much searching complexity is reduced. For simulations, we set $M_t = 2$. We assume that each element of $\mathbf{g_n}$ is generated by quantizing a random variable distributed as $\mathcal{CN} \sim (0, 0.1)$ into four levels, and the corresponding p_n is determined by integrating the probability density function over the associated quantization levels. The secondary transmit power ranges from 0 dB to 10dB. Accordingly, the MBIP problem has 16 binary design variables, such that if exhaustive search is deployed, there will be a total of $2^{16} = 65536$ sub-problems need to be solved. With our approach, Fig. 5 shows the update progress of the incumbent, and Fig. 6 shows the progress of pruned nodes in percentages at each iteration, where we only need to solve 273 sub-problems in this example.

Remark: The number of sub-problems solved in our BB algorithm varies over different channel realizations. Typically, we observe that less than 700 sub-problems in total are solved with our BB algorithm across a large number of channel realizations.

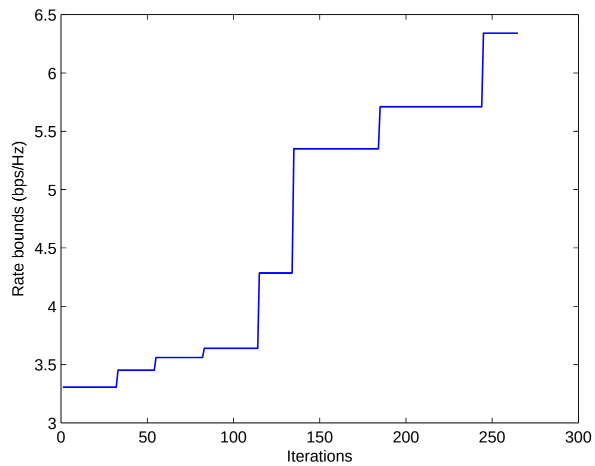

Fig. 5. Bounding progress of BB vs. the number of iterations.

4. Rank-One Property of the Optimal $\mathbf{K_x}$

Note that the authors in [13] studied a similar problem without the PU outage constraint, where they proved that the optimal $\mathbf{K_x}$ must be a rank-one matrix. To prove the rank-one property of the optimal matrix $\mathbf{K_x}$ in our case, we focus on the following

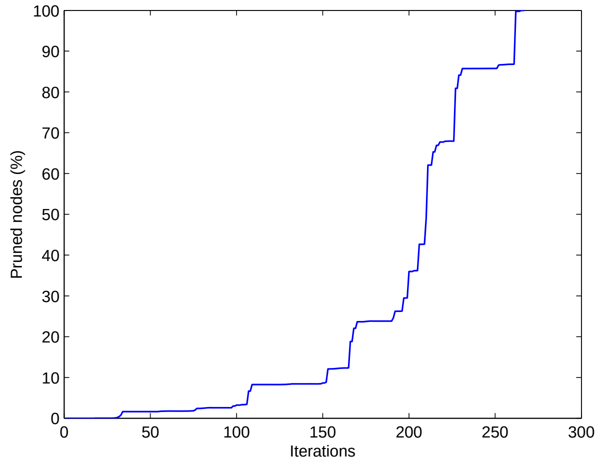

Fig. 6. Pruned nodes of BB vs. the number of iterations.

equivalent problem to the relaxed SDP problem at each given set of b_n's:

$$\begin{aligned}
\text{(P3)}: \quad \underset{\mathbf{K_x}}{\text{maximize:}} \quad & \log(1+\mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h}) && (3.19)\\
\text{subject to:} \quad & tr(\mathbf{K_x}) \leqslant P_{tr1} && (3.20)\\
& tr(\mathbf{K_x}E[\mathbf{g}\mathbf{g}^{\dagger}]) \leqslant P_{tr2} && (3.21)\\
& tr(\mathbf{K_x}\mathbf{g_n}\mathbf{g_n}^{\dagger}) \leqslant r, \quad \forall n \in T_1 && (3.22)\\
& tr(\mathbf{K_x}\mathbf{g_m}\mathbf{g_m}^{\dagger}) \leqslant r+M, \quad \forall m \in T_2 && (3.23)\\
& \mathbf{K_x} \succeq 0, && (3.24)
\end{aligned}$$

where the replacement of $\mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h}$ by $\log(1+\mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h})$ in the objective function is for the convenience of applying the Karush-Kuhn-Tucker (KKT) optimality conditions [29], the set T_1 contains all the indices with $b_n = 0$, and the set T_2 contains all the indices with $b_n = 1$. The Lagrange dual function of (P3) can thus be written as

$$\begin{aligned} g(\nu, \theta, \mu_n, \lambda_m) &= \sup_{\mathbf{K_x} \succeq \mathbf{0}} \; \log(1 + \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h}) \\ &+ \; tr\left[\mathbf{K_x}\left(\nu \mathbf{I} + \theta E[\mathbf{g}\mathbf{g}^\dagger] + \sum_{n \in T_1} \mu_n \mathbf{g_n}\mathbf{g_n}^\dagger + \sum_{m \in T_2} \lambda_m \mathbf{g_m}\mathbf{g_m}^\dagger\right)\right] \end{aligned} \quad (3.25)$$

where ν, θ, μ_n, and λ_m are the dual variables associated with the constraints (3.20) - (3.23), respectively. We then define matrix $\mathbf{A}$ as

$$\mathbf{A} = \nu \mathbf{I} + \theta E[\mathbf{g}\mathbf{g}^\dagger] + \sum_{n \in T_1} \mu_n \mathbf{g_n}\mathbf{g_n}^\dagger + \sum_{m \in T_2} \lambda_m \mathbf{g_m}\mathbf{g_m}^\dagger, \quad (3.26)$$

and show that $\mathbf{A}$ must have a full rank of M_t in order for the dual function to have a bounded value. First, it is clear that in the case of either $\nu > 0$ or $\theta > 0$, $\mathbf{A}$ must have full rank. When both $\nu = 0$ and $\theta = 0$, we prove that $\mathbf{A}$ also needs to have full rank by a contradiction approach discussed in [31]. Let us assume $\mathbf{A}$ is rank deficient; then it is possible to have a $\mathbf{K_x} = t\, \mathbf{v_j}\mathbf{v_j}^\dagger$, where $\mathbf{v_j}$ is an eigenvector of $\mathbf{A}$ corresponding to a zero eigenvalue and t is some scaling coefficient. As such, the trace term on the right-hand side of (3.25) goes to zero. Since $\mathbf{h}$ is drawn from a continuous distribution, the probability that $\mathbf{h}$ is orthogonal to $\mathbf{v_j}$ is zero. It thus follows that the supremum in (3.25) would be unbounded by choosing an appropriate polarity of t and scaling up the magnitude of t to infinity. As such, we conclude that $\mathbf{A}$ must have full rank, which allows us to define a new design variable $\overline{\mathbf{K}}_\mathbf{x} = \mathbf{A}^{\frac{1}{2}} \mathbf{K_x} \mathbf{A}^{\frac{1}{2}}$ and rewrite the Lagrange dual function as the optimal value of the following problem:

$$\underset{\overline{\mathbf{K}}_\mathbf{x}}{\text{maximize:}} \quad \log(1 + \mathbf{h}^\dagger \mathbf{A}^{-\frac{1}{2}} \overline{\mathbf{K}}_\mathbf{x} \mathbf{A}^{-\frac{1}{2}} \mathbf{h}) + tr(\overline{\mathbf{K}}_\mathbf{x}) \quad (3.27)$$

$$\text{subject to:} \quad \overline{\mathbf{K}}_\mathbf{x} \succeq 0. \quad (3.28)$$

This problem is convex and has strictly feasible points; thus the optimal $\overline{\mathbf{K}}_{\mathbf{x}}$ must satisfy the KKT conditions [29] as follows,

$$\frac{1}{\ln 2}(\mathbf{A}^{-\frac{1}{2}})^{\dagger}\mathbf{h}\left(1+\mathbf{h}^{\dagger}\mathbf{A}^{-\frac{1}{2}}\overline{\mathbf{K}}_{\mathbf{x}}\mathbf{A}^{-\frac{1}{2}}\mathbf{h}\right)^{-1}\mathbf{h}^{\dagger}(\mathbf{A}^{-\frac{1}{2}})^{\dagger}+\mathbf{\Phi} = -\mathbf{I} \qquad (3.29)$$

$$tr(\mathbf{\Phi}\overline{\mathbf{K}}_{\mathbf{x}}) = 0, \qquad (3.30)$$

where $\mathbf{\Phi} \succeq 0$ is the dual variable associated with the constraint (3.28). Here, we see that since the right-hand side of (3.29) is a matrix of full rank M_t, and the first term on the left-hand side has unit rank, the matrix $\mathbf{\Phi}$ must be of a rank greater than or equal to $M_t - 1$. Given $\overline{\mathbf{K}}_{\mathbf{x}} \succeq 0$ and $\mathbf{\Phi} \succeq 0$, together with (3.30), we conclude that the rank of the nontrivial optimal $\overline{\mathbf{K}}_{\mathbf{x}}$, and also the optimal $\mathbf{K}_{\mathbf{x}}$, is one. Since the above result holds for all of the feasible dual variables, when the ν, θ, μ_n and λ_m are taking the optimal values in the dual problem, the resulting optimal solution of $\mathbf{K}_{\mathbf{x}}$ from the optimal $\overline{\mathbf{K}}_{\mathbf{x}}$ in (3.27) - (3.28) is also the optimal solution for the original problem in (3.19) - (3.24), which is of rank one. As such, beamforming is optimal for the CR transmitter even under the PU outage probability constraint, where the optimal beamformer can be directly obtained as the eigenvector of the rank-one optimal $\mathbf{K}_{\mathbf{x}}$.

5. A Simple Heuristic Algorithm

As an alternative to the high-complexity BB-based solution, we propose a heuristic but efficient algorithm for finding a suboptimal solution of the MBIP problem (P2). By observing the objective function and the constraint (3.10) in (P2), we see that we will severely limit the SU received signal power when we limit the interference to the PU via (3.10) in the case of a $\mathbf{g_n}$ that is highly correlated with $\mathbf{h}$. To prevent this, we could manually set such a case as an outage scenario with $b_n = 1$ as long as the outage probability constraint is still satisfied. By doing so, the corresponding constraint $\mathbf{g_n}^{\dagger}\mathbf{K_x}\mathbf{g_n} \leq r + M$ becomes inactive with a large M, such that no power restriction is enforced over the correlated direction of $\mathbf{h}$. With the above approach applied to a part of $\mathbf{g_n}$'s correlated with $\mathbf{h}$, we could achieve a good balance between maximizing the SU rate and protecting the PU, where the philosophy is that since certain PU outage is allowed, we should greedily utilize such an outage allowance to cover certain $\mathbf{g_n}$'s that are aligned in a similar direction to $\mathbf{h}$.

Specifically, using the angles between $\mathbf{g_n}$'s and $\mathbf{h}$, defined by $\cos(\theta_n) = \frac{\bar{\mathbf{g}}_\mathbf{n}^\dagger \cdot \bar{\mathbf{h}}}{||\mathbf{g_n}||\ ||\mathbf{h}||}$, with $\bar{\mathbf{g}}_\mathbf{n} = [\mathrm{Re}(\mathbf{g_n})\ \mathrm{Im}(\mathbf{g_n})]^\dagger$ and $\bar{\mathbf{h}} = [\mathrm{Re}(\mathbf{h})\ \mathrm{Im}(\mathbf{h})]^\dagger$, as a measure for the correlation of directions, the proposed algorithm first sorts the set of $\{\mathbf{g_1}, \mathbf{g_2}, \cdots, \mathbf{g_N}\}$ in descending order of $|\cos(\theta_n)|$ and forms a new set $\{\tilde{\mathbf{g}}_\mathbf{1}, \tilde{\mathbf{g}}_\mathbf{2}, ..., \tilde{\mathbf{g}}_\mathbf{N}\}$ with a corresponding probability set $\{\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_N\}$. The $\tilde{b}_n$ values are all initialized to zeros. Starting with $\tilde{\mathbf{g}}_\mathbf{1}$, which has the highest correlation to $\mathbf{h}$ relative to other $\tilde{\mathbf{g}}_\mathbf{n}$'s, we add this scenario to the outage probability by setting the corresponding $\tilde{b}_1$ to one, as long as doing so does not violate the sum probability constraint $\sum_{n=1}^{N} \tilde{b}_n \tilde{p}_n \leqslant p_{th}$, otherwise $\tilde{b}_1$ is set to zero. This process continues sequentially for the set of $\{\tilde{\mathbf{g}}_\mathbf{1}, \tilde{\mathbf{g}}_\mathbf{2}, ..., \tilde{\mathbf{g}}_\mathbf{N}\}$, which results in a pre-determined set of $\tilde{b}_n$'s that satisfies the sum outage constraint and can be used to solve a convex SDP problem in the form of (P3). Though the optimality (with respect to the solution of (P2)) of the SU rate obtained by solving the above resulting problem is not guaranteed, it does offer an efficient solution to an otherwise complex problem. Numerical results in the next section shows the encouraging performance of this heuristic algorithm in comparison with the BB and reference algorithms. Note that the comment on the rank of $\mathbf{K_x}$ given in the last subsection is also applicable to this heuristic algorithm.

C. Numerical Results and Comparison

In this section, numerical results are presented to show the performance of the CR system under consideration with our optimal solution. The simulation setup is the same as that for generating Fig. 5 and Fig. 6. Fig. 7 illustrates the maximum

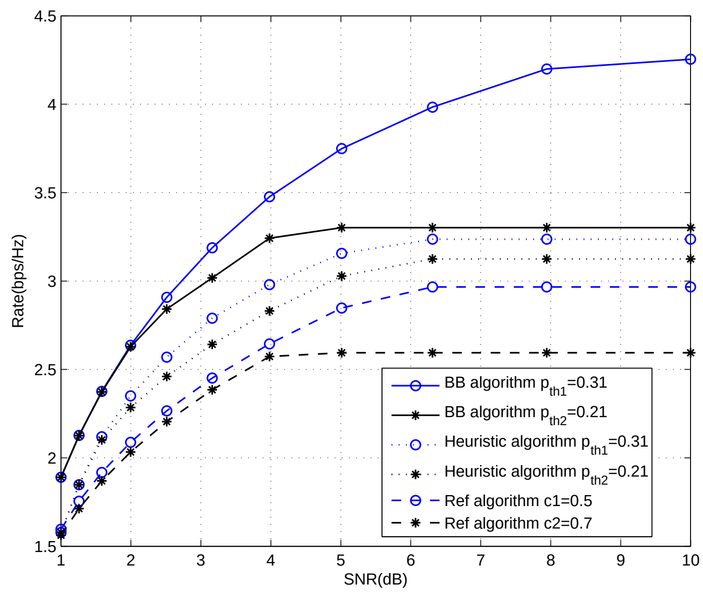

Fig. 7. Comparison of the achievable transmit rates with the BB algorithm, the heuristic algorithm, and the reference algorithm.

achievable transmit rate for the SU using the BB algorithm in comparison with the heuristic algorithm and a reference algorithm from [14] and [15]. In this case, we assume that the average interference power is limited to 2, and the outage probability limits are set as 0.21 and 0.31, respectively. The parameter values for c in the reference algorithm [14] are set to 0.7 and 0.5, which leads to outage probabilities of 0.21 and 0.31, respectively. From this figure, we see that the transmit rate with $p_{th} = 0.31$ is always greater than or equal to the rate with $p_{th} = 0.21$, which is as expected. Moreover, we note that the maximum achievable transmit rate with the BB approach is always larger than the rate of the heuristic approach, which is slightly larger than the reference algorithm.

CHAPTER IV

CONCLUSION

In this thesis, we considered a secondary communication link sharing the same spectrum with a primary link in a CR network. Multiple transmitting antennas are exploited at the SU-Tx to achieve balance between the SU transmit rate maximization and the interference regulation at the PU-Rx. We introduced the PU outage probability constraint in our formulation to model a more practical scenario, where the problem was formulated as a non-convex optimization problem with a probabilistic constraint, in addition to the SU transmit power constraint and a PU average interference constraint. To make the non-convex problem solvable, a deterministic transformation is used to approximate the original problem as an MBIP problem. An efficient BB algorithm and a simple heuristic algorithm were proposed to solve the MBIP problem, with simulation results to illustrate the superior performance of our algorithms over an existing reference algorithm. A key result proved in this paper is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under PU outage constraints. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

One natural extension for future work is to investigate a more general case with MIMO (instead of MISO) links between the SU-Tx and the SU-Rx along with multiple multi-antenna primary users.

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VITA

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